Chaotic hybrid new inflation in supergravity with a running spectral index

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We propose an inflation model in supergravity, in which chaotic and hybrid inflation occurs successively, followed by new inflation. During hybrid inflation, adiabatic fluctuations with a running spectral index with $n_s>1$ on a large scale and $n_s<1$ on a smaller scale are generated, as favored by recent results of the first year Wilkinson Microwave Anisotropy Probe. The initial condition of new inflation is also set dynamically during hybrid inflation, and its duration and the amplitude of density fluctuations take appropriate values to help early star formation to realize early reionization.

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I. INTRODUCTION

The recent results of the Wilkinson Microwave Anisotropy Probe (WMAP) confirmed the so-called concordance model, in which the Universe is spatially flat and has primordial adiabatic density fluctuations [1,2]. The spectrum of the density fluctuations is almost scale invariant but the running of the spectral index is favored from $n_s>1$ on a large scale to $n_s<1$ on a smaller scale.[1] More concretely, it is shown that $n_s=1.13\pm0.08$ and $dn_s/d\ln k=-0.055^{+0.028}_{-0.029}$ on the scale $k_0=0.002\ \mathrm{Mpc}^{-1}$ [5].

Though it is a nontrivial task to consider an inflation model with such a running spectral index, some models have been proposed mainly after the results of the WMAP have been released [6–11]. In the hybrid inflation model in supergravity proposed by Linde and Riotto [6], the running of the spectral index is obtained straightforwardly due to the contributions to the potential from both one-loop effects and supergravity effects. However, it is shown that its variation is too mild to explain the results of the WMAP [9]. This is mainly because the Yukawa coupling constant must be relatively small for sufficient inflation. To put it another way, a sufficient variation of the running may be obtained if another inflation follows the hybrid inflation of Linde and Riotto. Then, Kawasaki and the present authors considered a hybrid new inflation model in supergravity, which was originally proposed in [12–14], and showed that the results of the WMAP can be reproduced at the one-sigma level [9]. In this model, during hybrid inflation, primordial density fluctuations with such a running spectral index are generated and also the initial value of new inflation is set dynamically. If new inflation proceeds long enough, the scales with the desired spectral shape of density fluctuations can be pushed to cosmologically observable scales.

This model, however, has two drawbacks. First of all, hybrid inflation has a severe initial condition problem, that is, only very limited field configurations lead to inflation even if the field is assumed to be homogeneous [15]. Second, the density fluctuations generated at the onset of new inflation tends to be too large due to the uncontrollable smallness of the initial value of the scalar field responsible for new inflation. In the model discussed in [9], if we try to fit the central values of the spectral index and its running observed by WMAP, the scale l_* corresponding to the horizon at the onset of new inflation is larger than 100 kpc, so that too many dark-halo-like objects would be produced and cause cosmological problems. Hence we had to content ourselves with reproducing the WMAP data within one-sigma error in order to set l_* smaller than 1 kpc.

In this paper, we propose an improved model of inflation in supergravity, which resolves the above two problems. With regards to the former problem associated with initial conditions, the most natural scenario is chaotic inflation which occurs without any fine-tuning provided that the potential does not diverge too rapidly beyond the gravitational scale $M_G\simeq2.4\times10^{18}$ GeV [16]. This requirement, however, is difficult to achieve in supergravity and only a few successful models had been proposed in this context [17]. Recently, however, it was pointed out that such a large value can be realized by introducing the Nambu-Goldstone-like shift symmetry [18,19]. We make use of this symmetry and start with chaotic inflation. Furthermore, in our model, the initial condition for new inflation, which is set dynamically during hybrid inflation, is well under control and we can set it to satisfy various cosmological requirements with the central values obtained by WMAP, contrary to our previous model.

The rest of the paper is organized as follows. In the next section, we present our model of hybrid new inflation with a chaotic initial condition in supergravity. In Sec. III, we investigate the dynamics of hybrid inflation and show that primordial density fluctuations with a running spectral index are generated, as suggested by the results of the WMAP. In Sec. IV, we review new inflation in supergravity. In Sec. V, we investigate the dynamics of hybrid new inflation in supergravity. We show that the initial value of new inflation is set dynamically during hybrid inflation and the amplitude of

[1]Such a running of the spectral index is not yet confirmed definitely. The analyses which do not give the running feature of the spectral index are given, for example, in [3,4]. However, it is still important to give a concrete model with such a running spectral index because it is very difficult to realize such a feature in the context of a single-field inflation model.

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TABLE I. The symmetries and the charges of various superfields.

	X	S	Ψ	$\bar{\Psi}$	Z	Φ	Π	Σ	Ξ
Q_R	$\frac{n+2}{n+1}$	0	$\frac{n}{2(n+1)}$	$\frac{n}{2(n+1)}$	$-\frac{2}{n+1}$	$\frac{2}{n+1}$	$\frac{n}{2(n+1)}$	$\frac{3n+4}{2(n+1)}$	$\frac{n+2}{n+1}$
G	I	I	G	$\bar{G}$	I	I	I	I	I
Z_2	$+$	$-$	$+$	$+$	$+$	$+$	$-$	$-$	$-$

density fluctuations on the scale corresponding to the horizon at the onset of new inflation takes an appropriate value. Section VI is devoted to discussion and conclusion with particular emphasis on the fact that this model essentially involves only one energy scale.

II. MODEL

In this section, we give our model of chaotic hybrid new inflation in supergravity. In order to avoid blow up of the scalar potential such as $\exp(|S|^2/M_G^2)$ for the inflation, S, responsible for chaotic and hybrid inflation, we introduce the Nambu-Goldstone-like shift symmetry [18],

$$S \rightarrow S + iCM_G, \tag{1}$$

where C is a dimensionless real constant. As far as the symmetry is exact, however, the field S cannot have any potential. So we need to break it softly. By introducing a spurion superfield Ξ, we extend the shift symmetry such that the model is invariant under the following transformation [9]:

$$\begin{aligned} S &\rightarrow S + iCM_G, \\ \Xi &\rightarrow \frac{S}{S+iCM_G}\Xi. \end{aligned} \tag{2}$$

Then, the combination ΞS is invariant under the shift symmetry and the Kähler potential becomes a function of $S+S^*$, i.e., $K(S,S^*)=K(S+S^*)$, which allows the imaginary part of the scalar components of S to take a value much larger than M_G. If the spurion field acquires a vacuum expectation value, $\langle\Xi\rangle \ll M_G$, it softly breaks the above shift symmetry. Below we set M_G to unity and use the same notations for scalar fields and corresponding superfields.

We consider the following superpotential comprised of three parts:

$$\begin{aligned} W &= W_H + W_I + W_N, \\ W_H &= X(\lambda'\Psi\bar{\Psi} - \mu'^2\Pi^2) + g'\Xi\Psi\bar{\Psi} + \nu' Z\Xi^2 S^2, \\ W_I &= u'\Pi\Sigma Z\Phi, \\ W_N &= v'^2\Pi^4\Phi - \frac{h'}{n+1}\Phi^{n+1}. \end{aligned} \tag{3}$$

Here W_H induces chaotic and hybrid inflation, W_I determines the initial value of the new inflation, and W_N is a part associated with new inflation. In the above superpotential, Ψ and $\bar{\Psi}$ are a conjugate pair of superfields, which transform nontrivially under some gauge group G, while the other superfields are gauge singlet. In order to obtain such a superpotential, we introduce the $U(1)_R$ symmetry and the Z_2 symmetry, which are also broken softly by introducing the spurion fields Π and Σ, in addition to the shift symmetry. λ', μ', g', ν', u', v', and h' are coupling constants. The charge assignments for various superfields are shown in Table I. Note that all softly broken symmetries are restored when all the spurion fields have vanishing expectation values, $\langle\Xi\rangle=\langle\Pi\rangle=\langle\Sigma\rangle=0$. If Ξ acquires a nonvanishing expectation value, it breaks not only the shift symmetry but also the $U(1)_R$ symmetry and the Z_2 symmetry, so that we may expect that the magnitudes of the breaking of the three symmetries are mutually related. We will show later that even if we take a simple view that all the symmetry breaking scales are identical, $\langle\Xi\rangle=\langle\Pi\rangle=\langle\Sigma\rangle=\mathcal{O}(10^{-2})$, our model can reproduce various cosmological observations quite well with all the other model parameters being within a natural range of $10^{-3}\sim 1$. Then, by inserting the vacuum expectation values of the spurion fields and neglecting higher order terms, we have the superpotential[2]

$$\begin{aligned} W &= W_H + W_I + W_N, \\ W_H &= X(\lambda\Psi\bar{\Psi} - \mu^2) + gS\Psi\bar{\Psi} + \nu Z S^2, \\ W_I &= uZ\Phi, \\ W_N &= v^2\Phi - \frac{h}{n+1}\Phi^{n+1}. \end{aligned} \tag{4}$$

Here $\lambda \equiv \lambda' = \mathcal{O}(10^{-1})$, $\mu \equiv \mu'\langle\Pi\rangle = \mathcal{O}(10^{-3})$, $g \equiv g'\langle\Xi\rangle = \mathcal{O}(10^{-2})$, $\nu \equiv \nu'\langle\Xi\rangle^2 = \mathcal{O}(10^{-6})$, $u \equiv u'\langle\Pi\rangle\langle\Sigma\rangle = \mathcal{O}(10^{-7})$, $v \equiv v'\langle\Pi\rangle^2 = \mathcal{O}(10^{-6})$, and $h \equiv h' = \mathcal{O}(10^{-1})$.

On the other hand, the Kähler potential is given by[3]

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[2]Terms proportional to $\Xi S\Pi^2$ and ZX^2 can appear but here we have omitted them because they do not have significant effects on the dynamics, as can be easily shown.

[3]Terms associated with the breaking of the $U(1)_R$ symmetry can also appear but here we have omitted them because they do not have significant effects on the dynamics.

$$K = K_H + K_N + \cdots,$$

$$K_H = \frac{1}{2}(S+S^*)^2 + |X|^2 - \frac{c_X}{4}|X|^4 + |Z|^2 - \frac{c_Z}{4}|Z|^4 + |\Psi|^2 - \frac{c_\Psi}{4}|\Psi|^4 + |\bar{\Psi}|^2 - \frac{c_\Psi}{4}|\bar{\Psi}|^4, \quad (5)$$

$$K_N = |\Phi|^2 + \frac{c_N}{4}|\Phi|^4,$$

where c_X, c_Z, c_Ψ, and c_N are constants of the order of unity.

The potential of scalar components of the superfields z_i in supergravity is given by

$$V = e^K\left\{\left(\frac{\partial^2 K}{\partial z_i \partial z_j^*}\right)^{-1} D_{z_i} W D_{z_j^*} W^* - 3|W|^2\right\} + V_D, \quad (6)$$

with

$$D_{z_i} W = \frac{\partial W}{\partial z_i} + \frac{\partial K}{\partial z_i} W. \quad (7)$$

Here V_D represents the D-term contribution given by

$$V_D = \frac{e^2}{2}(|\Psi|^2 - |\bar{\Psi}|^2)^2, \quad (8)$$

in which we assume for simplicity that the gauge group G is U(1) and e is the gauge coupling constant. Then, the D-term contribution disappears for the direction $|\Psi| = |\bar{\Psi}|$.

III. HYBRID INFLATION FOLLOWING CHAOTIC INFLATION IN SUPERGRAVITY

In this section, we investigate the dynamics of hybrid inflation starting with a chaotic scenario. So, we concentrate only on W_H and K_H in the whole superpotential W and Kähler potential K. This treatment is justified because the energy scale of new inflation turns out to be much smaller than chaotic and hybrid inflation as will be seen later.

First of all, we decompose the scalar field S into real and imaginary components,

$$S = \frac{1}{\sqrt{2}}(\varphi + i\sigma). \quad (9)$$

Due to the Nambu-Goldstone-like shift symmetry, σ does not receive the exponential growth of the potential so that it can take a value much larger than unity under the chaotic initial condition [16]. Thus the potential is dominated by the quartic term of σ, that is,

$$V \simeq \frac{1}{4}\nu^2\sigma^4, \quad (10)$$

which leads to chaotic inflation. As σ rolls down along the potential, the false vacuum energy μ^4 becomes dominant and the usual hybrid inflation takes place successively.

During inflation, the mass terms of the other fields are given by

$$\begin{aligned} V \supset & \left(\frac{\nu^2}{4}\sigma^4 + \mu^4\right)\varphi^2 + \left(\frac{\nu^2}{4}\sigma^4 + c_X\mu^4\right)|X|^2 + \left(c_Z\frac{\nu^2}{4}\sigma^4 + \mu^4 + 2\nu^2\sigma^2\right)|Z|^2 + \left(\frac{\nu^2}{4}\sigma^4 + \mu^4 + \frac{g^2}{2}\sigma^2\right)(|\Psi|^2 + |\bar{\Psi}|^2) \\ & - \lambda\mu^2(\Psi\bar{\Psi} + \Psi^*\bar{\Psi}^*) - \frac{\nu}{2}\sigma^2\mu^2(XZ^* + X^*Z) \\ = & \left(\frac{\nu^2}{4}\sigma^4 + \mu^4\right)\varphi^2 + \left(\frac{\nu^2}{4}\sigma^4 + c_X\mu^4\right)\left|X - \frac{\frac{\nu}{2}\mu^2\sigma^2}{\frac{\nu^2}{4}\sigma^4 + c_X\mu^4}Z\right|^2 + \frac{|Z|^2}{\frac{\nu^2}{4}\sigma^4 + c_X\mu^4}\left(\frac{c_Z}{16}\nu^4\sigma^8 + \frac{1}{4}c_Xc_Z\mu^4\nu^2\sigma^4 + c_X\mu^8\right) \\ & + M_-^2|\Psi_+|^2 + M_+^2|\Psi_-|^2, \end{aligned} \quad (11)$$

where

$$M_\pm^2 = \pm\lambda\mu^2 + \frac{\nu^2}{4}\sigma^4 + \mu^4 + \frac{1}{2}g^2\sigma^2, \qquad \Psi_\pm = \frac{1}{\sqrt{2}}(\Psi \pm \bar{\Psi}^*). \quad (12)$$

Then, during hybrid inflation, the fields φ, X, Z, Ψ, and $\bar{\Psi}$ rapidly go to zero for $c_X, c_Z > 0$. M_-^2 becomes negative for $\sigma \simeq \pm\sigma_c$ with $\sigma_c \equiv \sqrt{2\lambda}\mu/g$ because $\nu^2\sigma_c^4/4, \mu^4 \ll \lambda\mu^2$. Therefore, at $\sigma \simeq \pm\sigma_c$, the phase transition takes place and hybrid inflation ends.

During hybrid inflation, the effective potential for σ is given by

$$V(\sigma)=\mu^4+\frac{\nu^2}{4}\sigma^4+V_{1L}. \tag{13}$$

Here V_{1L} is the one-loop correction given by [20]

$$V_{1L}=\frac{g^4}{128\pi^2}\left[(g^2\sigma^2+2\lambda\mu^2)^2\ln\frac{g^2\sigma^2+2\lambda\mu^2}{\Lambda^2}+(g^2\sigma^2-2\lambda\mu^2)^2\ln\frac{g^2\sigma^2-2\lambda\mu^2}{\Lambda^2}-2g^4\sigma^4\ln\frac{g^2\sigma^2}{\Lambda^2}\right], \tag{14}$$

where Λ is the renormalization scale. When $\sigma\gg\sigma_c$, it is approximated as

$$V_{1L}\cong\frac{\lambda^2\mu^4}{8\pi^2}\ln\frac{\sigma}{\sigma_c}. \tag{15}$$

Comparing the derivative of the second term and that of the last term in the effective potential (13), we find that the dynamics of the scalar field is dominated by the quartic term for $\sigma>\sigma_d\equiv\mu\sqrt{\lambda/(\sqrt{8}\pi\nu)}$ and by the radiative correction for $\sigma<\sigma_d$. Since $\nu\sim g^2$, σ_c and σ_d are comparable. Here we require $\sigma_c\lesssim\sigma_d(\ll 1)$.

The slow-roll parameters ϵ, η, and ξ are given by

$$\epsilon\equiv\frac{1}{2}\left(\frac{V'[\sigma]}{V[\sigma]}\right)^2\cong\frac{\nu^4}{2\mu^8}\sigma^6\left[1+\left(\frac{\sigma_d}{\sigma}\right)^4\right]^2=\mathcal{O}(\sigma^6),$$

$$\eta\equiv\frac{V''[\sigma]}{V[\sigma]}\cong\frac{V^2}{\mu^4}\sigma^2\left[3-\left(\frac{\sigma_d}{\sigma}\right)^4\right]=\mathcal{O}(\sigma^2), \tag{16}$$

$$\xi\equiv\frac{V'''[\sigma]V'[\sigma]}{V[\phi]^2}\cong\frac{2\nu^4}{\mu^8}\sigma^4\left[1+\left(\frac{\sigma_d}{\sigma}\right)^4\right]\left[3+\left(\frac{\sigma_d}{\sigma}\right)^4\right]=\mathcal{O}(\sigma^4).$$

On the other hand, the amplitude of curvature perturbation in the comoving gauge $\mathcal{R}$ [21] generated on the comoving scale $r=2\pi/k$ is given by

$$\mathcal{R}(k)=\frac{1}{2\pi}\frac{H^2(t_k)}{|\dot{\sigma}(t_k)|},\quad H^2(t_k)=\frac{V[\sigma(t_k)]}{3M_G^2}, \tag{17}$$

where t_k is the epoch when mode k left the Hubble radius during inflation [22]. The spectral index and its running are given by

$$n_s-1=-6\epsilon+2\eta\cong2\eta\cong\frac{2\nu^2}{\mu^4}\sigma^2\left[3-\left(\frac{\sigma_d}{\sigma}\right)^4\right],$$

$$\frac{dn_s}{d\ln k}=16\epsilon\eta-24\epsilon^2-2\xi\cong-2\xi\cong-\frac{4\nu^4}{\mu^8}\sigma^4\left[1+\left(\frac{\sigma_d}{\sigma}\right)^4\right]\left[3+\left(\frac{\sigma_d}{\sigma}\right)^4\right]. \tag{18}$$

σ and σ_d can be expressed by n_s-1 and $dn_s/d\ln k$ at $k=k_0$ as

$$\sigma=\sqrt{\frac{n_s-1}{2(3-q)}}\,\frac{\mu^2}{\nu},\qquad \sigma_d=q^{1/4}\sigma, \tag{19}$$

where

$$q\equiv\left(\frac{\sigma_d}{\sigma}\right)^4=\frac{3x+2\pm\sqrt{24x+1}}{x-1} \tag{20}$$

with

$$x\equiv-\frac{1}{(n_s-1)^2}\frac{dn_s}{d\ln k}.$$

Inserting the central values obtained by WMAPext+2dFGRS+$Ly\alpha$ on a comoving scale, $k_0=0.002\ \mathrm{Mpc}^{-1}$, $n_s-1=0.13$ and $dn_s/d\ln k=-0.055$ into the above equations, we find $\sigma\simeq0.19\mu^2/\nu$ and $\sigma_d\simeq0.21\mu^2/\nu$ with $\sigma_d/\sigma\simeq1.1$, which gives the relation

$$\frac{\mu^2}{\lambda\mu}\simeq2.7, \tag{21}$$

taking the lower sign in Eq. (20). Also, the amplitude of curvature perturbation in the comoving gauge $\mathcal{R}$ at σ is given by

$$\mathcal{R}(k_0)=\frac{1}{2\sqrt{3}\pi}\frac{\sigma\mu^6}{\nu^2(\sigma_d^4+\sigma^4)}\simeq5.6\nu=4.7\times10^{-5} \tag{22}$$

corresponding to $A=0.75$ of [5], which yields

$$\nu\simeq8.4\times10^{-6},\qquad \frac{\mu^2}{\lambda}\simeq2.2\times10^{-5}. \tag{23}$$

The e-folds of hybrid inflation after the comoving scale with the observed spectral shape has crossed the Hubble radius can be estimated as

$$N_H=\int_{\sigma_c}^{\sigma}\frac{V(\sigma)}{V'(\sigma)}d\sigma\simeq\frac{\mu^4}{\nu^2}\int_{\sigma_c}^{\sigma}\frac{\sigma}{\sigma_d^4+\sigma^4}d\sigma\simeq\frac{\mu^4}{\nu^2}\frac{1}{2\sigma_d^2}\arctan\left(\frac{\sigma^2}{\sigma_d^2}\right)\simeq8.6. \tag{24}$$

Thus another inflation must take place to push the relevant scale to the scale $k_0=0.002\ \mathrm{Mpc}^{-1}$.

Finally, from the constraint imposed on the amplitude of the tensor perturbations by WMAPext+2dFGRS+$Lx\alpha$ [5] we find an upper bound on the energy scale of hybrid inflation as

$$\mu<1.4\times10^{-2}, \tag{25}$$

corresponding to $H<1.1\times10^{-4}$.

IV. NEW INFLATION IN SUPERGRAVITY

In order to push the scales with the desired spectral shape of density fluctuations to cosmologically observable scales, we invoke new inflation which follows hybrid inflation discussed above. New inflation is also favorable in that it predicts low reheating temperature to avoid overproduction of gravitinos, contrary to hybrid inflation. Though new inflation has a severe initial condition problem in general, in our model, an appropriate initial condition for new inflation is dynamically realized during hybrid inflation. In this section, we briefly review new inflation induced by the superpotential W_N with the Kähler potential K_N [12].

The scalar potential derived from W_N and K_N is given by

$$V_N[\Phi]=\frac{\exp\left(|\Phi|^2+\frac{c_N}{4}|\Phi|^4\right)}{1+c_N|\Phi|^2}\times\left[\left|\left(1+|\Phi|^2+\frac{c_N}{2}|\Phi|^4\right)v^4 -\left(1+\frac{|\Phi|^2}{n+1}+\frac{c_N|\Phi|^4}{2(n+1)}\right)h\Phi^n\right|^2 -3(1+c_N|\Phi|^2)|\Phi|^2\left|v^2-\frac{h}{n+1}\Phi^n\right|^2\right]. \tag{26}$$

Then, the potential minimum is given by

$$|\Phi|_{\min}\cong\left(\frac{v^2}{h}\right)^{1/n}\quad\text{and}\quad \mathrm{Im}\,\Phi^n_{\min}=0, \tag{27}$$

with a negative energy density

$$V_N[\Phi_{\min}]\cong-3e^{K_N}|W_N[\Phi_{\min}]|^2\cong-3\left(\frac{n}{n+1}\right)^2v^4|\Phi_{\min}|^2. \tag{28}$$

Assuming that this negative value is canceled by a positive contribution due to supersymmetry breaking, $\Lambda^4_{\mathrm{SUSY}}$, we can relate energy scale of this model with the gravitino mass $m_{3/2}$ as

$$m_{3/2}\cong\frac{n}{n+1}\left(\frac{v^2}{h}\right)^{1/n}v^2. \tag{29}$$

Without loss of generality we may identify the real part of Φ with the inflation $\phi\equiv\sqrt{2}\,\mathrm{Re}\,\Phi$. The dynamics of inflation is governed by the lower-order potential

$$V_N[\phi]\cong v^4-\frac{c_N}{2}v^4\phi^2-\frac{2h}{2^{n/2}}v^2\phi^n+\frac{h^2}{2^n}\phi^{2n}. \tag{30}$$

Since the last term is negligible during inflation and the Hubble parameter is dominated by the first term, $H=v^2/\sqrt{3}$, the slow-roll equation of motion reads

$$3H\dot{\phi}=-V_N'[\phi]\cong-c_Nv^4\phi-2^{(2-n)/2}nhv^2\phi^{n-1}, \tag{31}$$

and the slow-roll parameters are given by

$$\epsilon\cong\frac{1}{2}\left(c_N\phi+2^{(2-n)/2}nh\frac{\phi^{n-1}}{v^2}\right)^2,$$

$$\eta=-c_N-2^{(2-n)/2}n(n-1)h\frac{\phi^{n-2}}{v^2}, \tag{32}$$

in this new inflation regime. Thus inflation is realized with $c_N\ll1$ and ends at

$$\phi=\sqrt{2}\left(\frac{(1-c_N)v^2}{hn(n-1)}\right)^{1/(n-2)}\equiv\phi_e, \tag{33}$$

when $|\eta|$ becomes as large as unity.

Since the two terms on the right-hand side of Eq. (31) are identical at

$$\phi=\sqrt{2}\left(\frac{c_Nv^2}{hn}\right)^{1/(n-2)}\equiv\phi_d, \tag{34}$$

the number of e-folds of new inflation is estimated as

$$N_N=-\int_{\phi_i}^{\phi_e}\frac{V_N[\phi]}{V_N'[\phi]}d\phi\cong\int_{\phi_i}^{\phi_d}\frac{d\phi}{c_N\phi}+\int_{\phi_d}^{\phi_e}\frac{2^{(n-2)/2}v^2}{hn\phi^{n-1}}d\phi =\frac{1}{c_N}\ln\frac{\phi_d}{\phi_i}+\frac{1-nc_N}{(n-2)c_N(1-c_N)}, \tag{35}$$

for $0<c_N<n^{-1}$. In the case that c_N vanishes, we find

$$N_N=\int_{\phi_i}^{\phi_e}\frac{2^{(n-2)/2}v^2}{hn\phi^{n-1}}d\phi=\frac{2^{(n-2)/2}v^2}{hn(n-2)}\phi_i^{2-n}-\frac{n-1}{n-2}. \tag{36}$$

Here ϕ_i is the initial value of ϕ, whose determination mechanism is discussed in the next section.

V. HYBRID NEW INFLATION IN SUPERGRAVITY WITH A CHAOTIC INITIAL CONDITION

In this section, we will investigate the whole dynamics of our model by considering the total superpotential (4) and the Kähler potential (5), under the condition

$$v^4\ll\mu^4. \tag{37}$$

Initially, the potential is dominated by the term $\nu^2\sigma^4/4$ so that inflation starts with the chaotic scenario. As inflation proceeds and the inflation σ falls to sub-Planckian scale, the false vacuum energy μ^4 becomes dominant so it turns to the usual hybrid inflation scenario. In this epoch, the mass terms of the other fields are given by

$$V\supset\left(\frac{v^2}{4}\sigma^4+\mu^4+v^4\right)\varphi^2+\left(\frac{\nu^2}{4}\sigma^4+c_X\mu^4+v^4\right)|X|^2+\left(c_Z\frac{\nu^2}{4}\sigma^4+\mu^4+v^4+2\nu^2\sigma^2\right)|Z|^2$$
$$+\left(\frac{\nu^2}{4}\sigma^4+\mu^4+v^4+\frac{g^2}{2}\sigma^2\right)(|\Psi|^2+|\overline{\Psi}|^2)+\left(\frac{\nu^2}{4}\sigma^4+\mu^4-c_Nv^4+u^2\right)|\Phi|^2-\lambda\mu^2(\Psi\overline{\Psi}+\Psi^*\overline{\Psi}^*)-\frac{\nu}{2}\sigma^2u(\Phi+\Phi^*)$$
$$+\mu^2v^2(X\Phi^*+X^*\Phi)+\frac{\nu}{2}\sigma^2v^2(Z\Phi^*+Z^*\Phi)-\frac{\nu}{2}\sigma^2\mu^2(XZ^*+X^*Z)$$
$$=\left(\frac{\nu^2}{4}\sigma^4+\mu^4+v^4\right)\varphi^2+M_-^2|\Psi_+|^2+M_+^2|\Psi_-|^2+\left(\frac{\nu^2}{4}\sigma^4+c_X\mu^4+v^4\right)|X|^2+\left(c_Z\frac{\nu^2}{4}\sigma^4+\mu^4+v^4+2\nu^2\sigma^2\right)|Z|^2$$
$$+\left(\frac{\nu^2}{4}\sigma^4+\mu^4-c_Nv^4+u^2\right)|\Phi|^2-\frac{\nu}{2}\sigma^2u(\Phi+\Phi^*)+\mu^2v^2(X\Phi^*+X^*\Phi)+\frac{\nu}{2}\sigma^2v^2(Z\Phi^*+Z^*\Phi)$$
$$-\frac{\nu}{2}\sigma^2\mu^2(XZ^*+X^*Z). \tag{38}$$

Clearly, the amplitudes of φ, Ψ, and $\overline{\Psi}$ rapidly vanish during hybrid inflation. When σ becomes equal to σ_c, M_-^2 becomes negative so that the phase transition occurs and hybrid inflation terminates. On the other hand, due to the presence of the term proportional to $\Phi+\Phi^*$, Φ deviates from the origin during inflation. Then, due to the cross terms with Φ, X and Z also deviate from the origin. The field values at the potential minimum can be estimated as

$$X_{\rm min}\simeq-\frac{4u\mu^2v^2}{\nu^3\sigma^6}\frac{1+c_Z}{c_Z},$$
$$Z_{\rm min}\simeq-\frac{2uv^2}{\nu^2\sigma^4}\frac{1}{c_Z}, \tag{39}$$
$$\Phi_{\rm min}\simeq\frac{u}{\nu\sigma^2},$$

for $\nu^2\sigma^4/4\gg\mu^4$, and

$$X_{\rm min}\simeq-\frac{u\nu\sigma^2v^2}{2\mu^6}\frac{1}{c_X},$$
$$Z_{\rm min}\simeq-\frac{u\nu^2\sigma^4v^2}{4\mu^8}\frac{1+c_X}{c_X}, \tag{40}$$
$$\Phi_{\rm min}\simeq\frac{u\nu\sigma^2}{2\mu^4},$$

for $\nu^2\sigma^4/4\ll\mu^4$. Here we have assumed that $2\nu^2\sigma^2<\mu^4$ in the mass squared of Z, which yields the constraint

$$2\sqrt{\lambda}\,\nu<g\mu. \tag{41}$$

It can be easily shown that the correction to the dynamics of hybrid inflation due to the presence of the deviations from the origins can be negligible if the following conditions are satisfied:

$$u\ll\mu^2,\quad \frac{\lambda uv^2}{g\mu^4}\ll1. \tag{42}$$

Then, all of the results derived in Sec. III still hold true.

Since the effective mass of the field $\phi=\sqrt{2}\,\mathrm{Re}\,\Phi$ is larger than the Hubble parameter during hybrid inflation, H_H, the above configuration is realized with the dispersion

$$\langle(\phi-\phi_{\rm min})^2\rangle=\langle\chi^2\rangle=\frac{3}{8\pi^2}\frac{H_H^4}{\mu^4}=\frac{\mu^4}{24\pi^2} \tag{43}$$

due to quantum fluctuations [23]. The ratio of quantum fluctuation to the expectation value should be less than unity,

$$\frac{\sqrt{\langle(\phi-\phi_{\rm min})^2\rangle}}{|\phi_{\rm min}|}=\frac{1}{4\sqrt{3}\pi}\frac{g^2\mu^4}{u\nu\lambda}\ll1, \tag{44}$$

where we have used

$$\phi_{\rm min}\simeq\frac{u\nu\sigma_c^2}{\sqrt{2}\mu^4}\simeq\frac{\sqrt{2}u\nu\lambda}{g^2\mu^2}, \tag{45}$$

corresponding to the value at the end of hybrid inflation, $\sigma\simeq\sigma_c$. Inserting the values $\nu\simeq8.1\times10^{-6}$ and $\mu^2/\lambda\simeq2.2\times10^{-5}$ yields the constraint

$$\lambda h\ll6.3\times10^4, \tag{46}$$

which is trivially satisfied. Thus the initial value of the inflation for new inflation is located off the origin with an appropriate magnitude.

After hybrid inflation ends, ϕ oscillates and its amplitude decreases with an extra factor v/μ by the time vacuum energy density v^4 dominates the total energy density [13], so new inflation starts with

$$\phi_i=\frac{\sqrt{2}u\nu\lambda v}{g^2\mu^3}, \tag{47}$$

and continues until $\phi=\phi_e$ with the potential (30).

TABLE II. List of model parameters. The expectation values of the spurion fields are taken to be mutually identical: $\langle\Xi\rangle=\langle\Pi\rangle=\langle\Sigma\rangle=10^{-2}B$.

Parameter	Origin	Order of magnitude	Role
μ	$\mu'\langle\Pi\rangle$	10^{-3}	energy scale of hybrid inflation
v	$v'\langle\Pi\rangle^2$	10^{-6}	energy scale of new inflation
u	$u'\langle\Pi\rangle\langle\Sigma\rangle$	10^{-7}	interaction between hybrid inflaton and new inflaton
ν	$\nu'\langle\Sigma\rangle^2$	10^{-6}	square root of the self-coupling of chaotic inflaton
λ	λ'	0.1	tachyonic mass parameter at the end of hybrid inflation
g	$g'\langle\Xi\rangle$	10^{-2}	mass of hybrid inflaton
h	h'	0.1	self-coupling of new inflaton

Contrary to the hybrid inflation regime, we do not have significant observational constraints on the new inflation regime. As stated in the Introduction, generally speaking, the density fluctuations generated during new inflation can become large because of the smallness of the field value of the inflation. However, since hybrid inflation does not last so long, the comoving scale which leaves the horizon at the beginning of new inflation is larger than 100 kpc so that the density fluctuations on the corresponding scales should take an appropriate value for structure formation, which is easy to realize in this model, contrary to our previous model [9].

For definiteness, we consider the case with $n=4$ and $c_N=0$. Then, from Eq. (36), the number of e-folds of new inflation reads

$$N_N=\frac{v^2}{4h}\frac{1}{\phi_i^2}-\frac{3}{2}. \tag{48}$$

This should be around 40 to push the comoving scale with appropriate spectral shape to the appropriate physical length scale,[4] which yields the relation

$$\frac{u}{g^2\sqrt{\lambda}}\simeq\frac{6.9\times10^{-4}}{\sqrt{h}}. \tag{49}$$

Then the spectral index at the onset of new inflation, $\phi=\phi_i$, reads

$$n_s-1=-6\epsilon+2\eta\cong-12h^2\frac{\phi_i^6}{v^4}-12h\frac{\phi_i^2}{v^2}\cong2\eta\cong-0.07. \tag{50}$$

On the other hand, the amplitude of curvature perturbation at the same scale is given by

$$\mathcal{R}\simeq\frac{v^4}{4\sqrt{3}\pi h\phi_i^3}\simeq98\sqrt{h}v\simeq10^{-4}. \tag{51}$$

Here we have required $\mathcal{R}\simeq10^{-4}$.

Now we have listed all the constraints on the model parameters (23), (25), (37), (41), (42), (46), (49), and (51). As listed in Table II, our model has seven parameters after the spurion fields have acquired expectation values. Among them, ν is fixed by the normalization of fluctuation amplitude. We have three more equalities, (23), (49), and (51), for the other six undetermined parameters, λ, g, h, μ, v, and u. Hence we can express all these parameters in terms of three independent variables. That is, if we fix the expectation values of the spurion fields, we can describe all the parameters of the theory as functions of g', μ', and v'. To do this let us take a simple view that the expectation values of the spurions are all equal and normalize them by 10^{-2}, that is,

$$\langle\Xi\rangle=\langle\Pi\rangle=\langle\Sigma\rangle\equiv10^{-2}B, \tag{52}$$

where B is a parameter of order of unity. Then by definition we find

$$g=10^{-2}g'B,\quad \mu=10^{-2}\mu'B,\quad \text{and } v=10^{-4}v'B. \tag{53}$$

From Eqs. (23), (49), and (51), other parameters in Eq. (4) read

$$\begin{aligned}&\lambda\simeq4.4\mu'^2B^2,\quad h\simeq1.0\times10^{-4}v'^{-2}B^{-4},\\&u=10^{-4}u'B^2\simeq1.4\times10^{-5}g'^2\mu'v'B^5,\\&\nu=10^{-4}\nu'B^2\simeq8.4\times10^{-6}.\end{aligned} \tag{54}$$

Parameters in the original superpotential are given by

$$\begin{aligned}&\lambda'\simeq4.4\mu'^2B^2,\quad h'\simeq1.0\times10^{-4}v'^{-2}B^{-4},\\&u'\simeq0.14g'^2\mu'v'B^3,\quad \nu'\simeq8.4\times10^{-2}B^{-2}.\end{aligned} \tag{55}$$

From Eqs. (25), (37), (41), (42), and (46), we find that g', μ', and v' must satisfy

[4]Strictly speaking, extra e-folds $\ln(\mu/v)$ should be added in making a correspondence between comoving horizon scales during hybrid inflation and proper scales. This is because comoving scales that left the Hubble radius in the late stage of hybrid inflation reenter the horizon before the beginning of the new inflation. However, for simplicity, we set $N_N\simeq40$.

$$\mu' < 1.4B^{-1}, \quad \mu'^{-1}v' \ll 10^2 B^{-1}, \quad g' > 0.36B^{-1},$$
$$g'^2\mu'^{-1}v' \ll 7.0B^{-3}, \quad g'\mu'^{-1}v'^3 \ll 1.6\times 10^2 B^{-6}, \tag{56}$$
$$\mu' v'^{-1} \ll 1.2\times 10^4 B.$$

If these inequalities are satisfied, which are in fact easily satisfied with natural magnitudes of model parameters, our model is viable. As an example, let us take $B=1$, $g'=1$, $\mu'=0.3$, and $v'=0.02$, which yields

$$\lambda=0.40, \ h=0.25, \ g=10^{-2}, \ \mu=3.0\times 10^{-3},$$
$$v=2\times 10^{-6}, \ u=8.6\times 10^{-8}, \tag{57}$$

with

$$\lambda'=0.40, \ h'=0.25, \ u'=8.6\times 10^{-4}, \ \text{and} \ \nu'=8.4\times 10^{-2}. \tag{58}$$

In this case the gravitino mass takes an acceptable value, $m_{3/2}=15$ TeV.

VI. DISCUSSION AND CONCLUSION

In this paper, we proposed an inflation model in supergravity, in which hybrid inflation starts with a chaotic initial condition and new inflation follows hybrid inflation. In order to realize chaotic inflation in supergravity, we introduced the Nambu-Goldstone-like shift symmetry, which ensures the flatness beyond the gravitational scale M_G. During hybrid inflation, adiabatic fluctuations with a running spectral index with $n_s > 1$ on a large scale and $n_s < 1$ on a smaller scale are generated, as inferred by the first-year data of WMAP. The initial condition of new inflation is also set dynamically during hybrid inflation and we can acquire a sufficiently large number of *e*-folds with the amplitude of fluctuation at its onset well under control.

The latter feature is especially important, because, if the amplitude of fluctuation at the onset of new inflation, corresponding to ~100 kpc today, turned out to be too large as in our previous model [9], the Universe would be too clumpy on this scale with too many dark-halo-like objects. On the other hand, if we extrapolate the best-fit spectrum obtained by the first-year WMAP data with a running spectral index to smaller scales, galactic and smaller scale fluctuations tend to be too small to realize timely galaxy formation. In our model the amplitude of fluctuations generated during new inflation can easily take appropriately larger values than that obtained by simply extrapolating the large-scale power spectrum. This may be helpful for early star formation which is required for early reionization [1] and from the age estimate of high-redshift quasars using the cosmological chemical clock [24].

Finally we comment on the naturalness of our model. Since our model realizes three different inflationary scenarios in succession, its Lagrangian is inevitably more complicated than that of a simple single field model. In particular, since the final form of the superpotential contains seven parameters, whose order of magnitude ranges from 10^{-7} to 1, and the energy scales of the latter two inflation are strikingly different from each other, one may wonder if it is too complicated and *ad hoc*. If we return to a more fundamental level of the theory with spurion fields, however, we find that all the coupling parameters take values within a natural range of $10^{-3}\sim 1$, and that it contains only a single energy scale, namely, the expectation values of the spurion fields can be mutually identified with $\mathcal{O}(10^{-2})$, a typical unification scale. At the present level of our understanding of the fundamental theory, we may only say that this scale is determined by cosmological observation, namely, the amplitude of density fluctuations, as with simpler models of single-field inflation, whose energy scale is usually determined by the same observation. Thus our model essentially has only one energy scale and the huge difference of the energy scale between hybrid inflation and subsequent new inflation is naturally realized by virtue of the symmetries of the theory.

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